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Frozen Rigging Model of the Energy Dominated Universe

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Composite rigging systems, involving membranes that meet on strings that meet on monopoles, arise naturally by the Kibble mechanism as topological defects in field theories involving spontaneous symmetry breaking. Such systems will tend to freeze out into static lattice type configurations with energy contribution ultimately be provided by the membranes. It has been suggested by Bucher and Spergel that on scales large compared with the relevant (interstellar separation) distance characterizing the relevant mesh length, such a system may behave as a rigidity-stabilized solid, having an approximately isotropic stress energy tensor with negative pressure, as given by a polytropic index $\gamma = w + 1 = 1/3$. It has recently been shown that such a system can be rigid enough to be stable if the number of membranes meeting at a junction is even (though not if it is odd). Using as examples an approximately O(3) symmetric scalar field model that can provide an "8-color" (body-centered) cubic lattice, and an approximate $U(1)\times U(1)$ model offering a disordered "5-color" lattice, it is argued that such a mechanism can account naturally for the observed dark energy dominance of the universe, without ad hoc assumptions, other than that the relevant symmetry breaking phase transition should have occurred somewhere about the Kev energy range.

KEY WORDS: cosmology; elasticity.

1. INTRODUCTION

Kibble's idea that p-branes—meaning structures with support confined to a thin neighborhood of a $(p + 1)$-dimensional worldsheet—may form naturally, as vacuum analogues (in field theories involving spontaneous symmetry breaking) of the kind of topological defect that is familiar in condensed matter physics (Kleman, 1995), has attracted widespread interest in recent years (Kibble, 1995), particularly in the context of cosmology, for which the consequences may be important. The most commonly discussed possibility is that of cosmic string formation during Grand Unification, but there are many other eventualities whereby other later lower energy phase transitions could have given rise to vacuum defects of many other kinds. As well as scenarios giving rise to isolated monopoles or simple (domain wall type) membranes, there is a rich variety of more complex possibilities, such

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as strings with monopole endpoints or intersections, and membranes with string boundaries or junctions. Furthermore the strings and membranes involved need not necessarily be of simple Dirac–Goto–Nambu kind—with action just proportional to their worldsheet measure giving a tension T say that is constant and equal (in relativistic units) to the mass density—but might also be of more complicated kind with variable (and in the membrane case anisotropic) tension due to internal currents of the kind first suggested by Witten in the case of cosmic strings, whose loops can thereby be stabilized as vortons (Davis and Shellard, 1989).

Convenient mathematical machinery is now available (Carter, 1995) for treating the dynamical evolution, at a local level, of a system of such p-branes provided they behave collectively as a "regular rigging" structure (using terminology suggested by the nautical analogy with a system of ropes and sails) in the sense that each p-brane worldsheet interacts only by direct contact with $p-1$ branes forming its boundary or $p+1$ branes of which it is itself a boundary. A prototypical example of the application of this machinery is illustrated (Carter *et al.*, 2002) by the case of drum vortons. However, as in the familiar case of point particles (i.e., zero-branes) so also more generally, in the thin brane limit, technical problems will turn up due to divergences if it is necessary to allow for interactions between branes with dimension differing by 2 or more. In particular there will be divergence problems whenever it is necessary to allow for long-range self-interaction via the intermediary of electromagnetic and gravitational fields, whose support is the underlying four-dimensional spacetime manifold, which, considered as a continuous medium, is effectively a 3-brane.

Such an underlying 3-brane can interact in a well-behaved manner (as described by Darmois-Israel type (Battye and Carter, 2001) junction conditions) with an ordinary membrane, which counts as a 2-brane, but not with strings and point particles. Provided that (as will usually be the case for the topological defects in question) the coupling of the long-range self-interactions is weak enough for a linearized approximation to suffice, the effect of the divergences can be allowed for at a local level in point particles, and to some extent in strings (Carter and Battye, 1998; Carter *et al.*, 2003) by appropriate regularization and renormalization procedures. However, a satisfactory treatment of the finite adjustment terms needed to allow for the effect of radiation back-reaction in strings remains elusive—even for scalar axion radiation and linear electromagnetic radiation, and a fortiori for the gravitational radiation whose effects are important (Shellard, 1995) for the long-term evolution of a heavy string distribution.

Issues concerning heavy strings have in recent years come to seem less important than previously, since trends in observational cosmology now disfavor the idea that GUT strings played a decisive role in galaxy formation. Attention has been focussed instead on a new issue which is that of the evidence for large-scale cosmic acceleration implying the presence of a predominant "dark energy" constituent with a negative mean pressure P. The required value is so large—compared with the apparent mass density ρ, which itself includes a substantial dark matter contribution—that doubt has even been cast (Carroll *et al.*, 2003) on such a generally accepted assumption as the "dominant energy condition" of Hawking and Ellis, on which the demonstration of their (classical) vacuum conservation theorem is based (Carter, 2003). Although this need not apply by quantum fluctuations on a microscopic scale, it is indeed hard to see how a catastrophic instability of the vacuum could be avoided in a theory violating this condition, to the effect that at a macroscopic average level,

$$\rho \geq |P| \,. \tag{1}$$

The present situation (Conversi *et al.*, 2004) is that the available evidence does not seem to impose quite such a radically revolutionary step as abandoning the usual supposition (1), nor even to impose the marginal limit $\rho = |P| = -P$ requiring invocation of a cosmological constant (whose magnitude would be hard to account for except by anthropic considerations (Garriga *et al.*, 2004)). What it does seem to call for, rather conclusively, is the abandoning of the more restrictive condition $P > 0$ that is usually postulated as a necessity for avoiding microscopic instability in a perfect fluid model.

2. THE NEED FOR A SOLID MODEL

In a perfect fluid model that is barotropic, meaning that its pressure P is a function just of its mass–energy density ρ, what is actually needed for avoiding microscopic instability is that there should be a real value for the sound speed c_s as given (even in a relativistic model) by Newton's well-known formula

$$c_s^2 = \frac{dP}{d\rho} \,. \tag{2}$$

In the particular case of a relativistic polytrope, meaning a model for which the density is proportional to a power of a conserved number density n say, i.e., $\rho \propto n^\gamma$ where γ is the polytropic index (whose value in the familiar example of electromagnetic black-body radiation is $\gamma = 4/3$) we shall have

$$P = w\rho \,, \qquad c_s^2 = w \,, \tag{3}$$

in terms of the so-called "equation-of-state parameter" w as defined in terms of the polytropic index by $w = \gamma - 1$. A negative value of $dP/d\rho$ would imply an imaginary value of c_s, which would be interpretable as meaning that short wavelength perturbations would undergo rapid exponential growth. The condition for avoidance of such instability is evidently expressible as

$$\frac{dP}{d\rho} > 0 \;\; \Rightarrow \;\; w > 0 \,, \tag{4}$$

which in conjunction with the energy condition (1) is equivalent to the usual supposition $P > 0$.

Since the condition (4) seems (Conversi *et al.*, 2004) to be inconsistent with the available cosmological evidence, the conclusion to be drawn is that we need a model of a kind more general than a polytropic fluid. Although it is hard to see how to construct a viable theoretical model that violates the energy dominance postulate (1), it was pointed out by Bucher and Spergel (1999) that it is easy to construct a continuum model that gets round the restriction (4) if, instead of assuming that it behaves as a fluid, one supposes that it will behave as a solid with a sufficiently large rigidity modulus μ. Generalizing results that are well known in non-relativistic elasticity theory, it was shown many years ago by the present author (Carter, 1973) that, in a relativistic elastic solid, the speed, $c_{\parallel}$ say, of longitudinally polarized propagation modes will be given in terms of the value that it would have according to (2) in the absence of rigidity by

$$c_{\parallel}^2 = c_{s}^2 + \frac{4}{3}\, c_{\perp}^2\,, \tag{5}$$

where $c_{\perp}$ is the speed of transversely polarized (shake type) modes, which will be given by

$$c_{\perp}^2 = \frac{\mu}{\rho + P}\,. \tag{6}$$

It is evident that these speeds will both be real, and hence that the isotropic solid state will be locally stable, provided not only that the rigidity is positive, $\mu > 0$, but that it also satisfies

$$\frac{\mu}{\rho} > -\gamma\, w\,, \tag{7}$$

which is evidently a more restrictive requirement if $w < 0$. It can be seen that this stability criterion will be satisfied for all (negative as well as positive) values of w if

$$\frac{\mu}{\rho} > \frac{1}{4}\,. \tag{8}$$

3. MEMBRANES VERSUS STRINGS

Having presented the case in favor of a solid model, Bucher and Spergel went on to suggest (Bucher and Spergel, 1999; Battye *et al.*, 1999) that a medium of this kind might arise naturally as a large-scale average representation of a distribution of approximately static cosmic strings of the simple Nambu Goto type with tension T equal to energy density, for which the effect of three-dimensional averaging would give an effective average tension, meaning a negative pressure, that would be one-third of the average density, i.e.,

$$w = -1/3 \quad \Leftrightarrow \quad \gamma = 2/3\,. \tag{9}$$

They also effectively resuscitated an earlier idea (first put forward in a very different astrophysical context when the cosmological evidence for dark energy dominance was not so strong) of Kubotani and collaborators (Ishahara *et al.*, 1992; Den *et al.*, 2000) by pointing out that from the point-of-view of the cosmological evidence a more satisfactory agreement would be provided by a distribution of Dirac type membranes, for which the tension would also be equal to the energy but for which averaging over space directions would give a mean tension twice as large, so that one would have

$$w = -2/3 \quad \Leftrightarrow \quad \gamma = 1/3\,. \tag{10}$$

As when a gas of particles is treated as a fluid, the continuum description of such a distribution of strings or membranes as a coherent medium would presumably be valid only at a macroscopic level sufficiently large compared with some relevant interaction lengthscale. The question of whether the elastic deformation energy of such an isotropic string or membrane distribution really would be sufficient to satisfy the stability condition (7) was not checked until very recently, but the result (Battye *et al.*, 2005) turns out to be positive, at least for systems of even type, meaning those for which the branching number at junctions is even, so that the intersections can be described in terms of crossing without deflection—with the implication that local equilibrium of a distribution of straight strings or plane membranes can be preserved by linear deformation. It has been found for such cases that the effective rigidity modulus describing the large-scale averaged value of such a distribution will be the same for the membrane case as for the string case, having a value given by

$$\frac{\mu}{\rho} = \frac{4}{15}\,. \tag{11}$$

This is clearly sufficient—though not by a very large margin—to satisfy the general stability condition (8), a consideration that is important in view of the likelihood that—even if it is dominated by membranes—a naturally occurring defect distribution would be likely to include a certain proportion of strings (in particular those forming the junctions between membranes) and therefore to be characterised by an effective index in the intermediate range $-2/3 < w < -1/3$, $1/3 < \gamma < 2/3$.

The traditional simplified Friedman model of a homogeneous isotropic universe is governed by an evolution equation expressible (in Planck units with $c = G = \hbar = 1$) in the well-known form

$$H^2 = \frac{4\pi}{3}\rho\,, \tag{12}$$

where H is the Hubble expansion rate as given with respect to the proper time t in terms any co-moving length scale λ by $H = \lambda^{-1} d\lambda/dt$. The preceding considerations suggest that in addition to the usual cold matter contribution proportional to a conserved number density

$$n = \lambda^{-3}\,, \tag{13}$$

say, and the (in the early stages dominant) black-body radiation contribution proportional to $n^{4/3} = \lambda^{-4}$, we should add in an allowance for a cosmic string contribution proportional to $n^{2/3} = \lambda^{-2}$ and a cosmic membrane contribution to $n^{1/3} = \lambda^{-1}$. (In a brane world cosmological scenario one would also need extra terms (Binétruy *et al.*, 2000; Carter and Uzan, 2001) to allow for effects of higher dimensions that could have been important at very early stages in the evolution, but this should not be necessary for the present purpose which is to consider the less speculative question of the more recent evolution, starting not too much before the observationally accessible era at which primordial element formation took place).

Under these assumptions the equation of state for the total mass density will be expressible in the convenient form

$$\rho = \frac{3}{4\pi}\left(n^{4/3} + a\,n + b\,n^{2/3} + c\,n^{1/3}\right) \tag{14}$$

in terms of just three constant coefficients, a, b, c, of which the first drops out in the corresponding expression for the pressure, namely

$$P = \frac{1}{4\pi}\left(n^{4/3} - b\,n^{2/3} - 2c\,n^{1/3}\right)\,. \tag{15}$$

The calibration adopted here allows the Hubble equation to be written in the simple explicit form

$$\frac{dt}{d\lambda} = \frac{\lambda}{\sqrt{1 + a\,\lambda + b\,\lambda^2 + c\,\lambda^3}}\,. \tag{16}$$

These expressions implicitly fix the normalization of the co-moving lengthscales λ, which can be seen to be interpretable as a mean black-body radiation wavelength, i.e., $\lambda \approx 2\pi\Theta^{-1}$ where Θ is the temperature, whose present day value, Θ_c say, corresponds to a wavelength in the millimeter range, which in Planck units means something like $\Theta_c \approx 10^{-31}$. The parameter a therefore represents the temperature at which cold matter and radiation densities were comparable, which appears to be somewhat less than the Rydberg recombination energy, (the exact value depending on how many neutrino species are supposed to be present) so that in terms of the fine structure constant $e^2 \approx 1/137$ and the electron mass $m_e \approx 10^{-22}$ it will be given very roughly by $a \approx 10^{-1} e^4\, m_e \approx 10^{-27}$.

The novelty in the model considered here is the introduction of the other two coefficients b and c of which the latter, namely the membrane density coefficient c, is presumably the most important. The string density coefficient b is likely to be relatively unimportant, because it is expected that the string contribution would have been dominated by the cold matter and radiation contributions at earlier stages, while it will evidently be dominated by the membrane contribution at later stages, and already—if observational appearances are to be believed—even at the present epoch. In order for the membrane term to be comparable with the cold matter term as from about now, it is clearly necessary that its coefficient should be given in rough order of magnitude by $c \approx a\,\Theta_c^2 \approx 10^{-89}$.

4. MEMBRANE FORMATION BY THE KIBBLE MECHANISM

Up to this stage, the line of logic presented here has followed the principles originally proposed by Bucher and Spergel (1999; 1999), but—in so far as the mechanism governing the creation of the membranes, and the implications for the value of the membrane density coefficient c as a function of the membrane tension, T, is concerned—I now have to express a somewhat dissident opinion. I agree that it is reasonable to assume that, after the cosmological temperature Θ had fallen below the value, η say, characterizing the symmetry breaking phase transition by which they were formed, the membranes would have rapidly settled down "to an equilibrium distribution which is swept along by the Hubble flow." However, I do not agree with the supposition, expressed in the same sentence (Battye *et al.*, 1999), that there would initially have been "one wall per horizon volume," i.e., the minimum that is causally conceivable. The reasoning below would have it that the correct value should be not one but more like 10^{10}.

A supposition to the effect that there is initially a single defect per horizon volume is of course the usual starting point for studies (Shellard, 1995) of free oscillations, radiation damping, et cetera, but only after, not before, the Kibble transition at which the assumption that the distribution is frozen into the Hubble flow ceases to be valid. For defects formed when $\Theta \approx \eta$ it is expected (Vilenkin and Shellard, 1994) that the Kibble de-freezing transition will occur when $\Theta \approx \eta^2$, which will happen at a fairly early stage for very heavy defects such as GUT strings (as characterised by $\eta \approx 10^{-3}$). However for the much lower values of η that are relevant in the present context, the temperature will still be much too high, even at the present epoch, for the Kibble transition to have occurred at all. This simplifies the analysis in many ways—justifying the neglect of computationally awkward effects such as gravitational radiation reaction, and allowing us to assume with confidence, at least as a first approximation, that the distribution will indeed be "swept along by the Hibble flow"—but it also means that it is quite inappropriate to assume that there was initially just "one wall per horizon volume."

The correct supposition, or at any rate what is usually assumed in discussions (Vilenkin and Shellard, 1994) of Kibble's defect formation mechanism, is that there would indeed have initially been one defect per correlation volume, but that this volume would have been the cube of a correlation length, ξ say, that would initially have been much smaller the horizon length scale H^{-1} (a causal limit value that it can attain only at a much later stage, if and when the Kibble transition temperature is reached). The usual Kibble ansatz (Carter *et al.*, 2000) (based on random walk considerations) for the initial value of the relevant correlation length is that it would have been the geometric mean of the horizon scale H^{-1} and the thermal lengthscale, $\lambda \approx \Theta^{-1}$ at the epoch of the phase transition, which gives

$$\xi \approx \eta^{-3/2}\,, \tag{17}$$

(whereas the ansatz used by Bucher and Spergel amounted to taking the much larger initial value $\xi \approx \eta^{-2}$).

In the simple cosmic string case the subsequent increase in the correlation scale (which eventually catches up with the Hubble radius at the Kibble transition) will entail erasure of structure on smaller scales (small wiggles will be damped out and small loops will contract to nothing) but in the kind of membrane network envisaged here the damping process characterised by the expanding lengthscale ξ will not entirely destroy the structure on smaller scales but merely freeze it into a configuration with locally minimized energy, which one would expect to resemble a crystalline or glass-like lattice of cellular vacuum domains characterised by a "rigging" lengthscale ℓ say. This rigging lengthscale would initially be the same as the damping lengthscale ξ but it would expand more slowly, merely being dragged along by the Hubble flow and therefore given at later times by

$$\ell \approx \eta^{-1/2}\Theta^{-1}\,. \tag{18}$$

(This is to be compared with the corresponding formula for the evolution of the damping scale, which according to the usual analysis (Vilenkin and Shellard, 1994) will be given by $\xi \approx \eta\,\Theta^{-5/2}$).

Since the surface density is given by the membrane tension T, the mass–energy associated with a single wall of a single cellular vacuum domain will given in order of magnitude by $T\,\ell^2$. This leads to an expression of the form

$$\rho \approx \frac{T}{\ell}\,. \tag{19}$$

for contribution of the membrane distribution to the cosmic mass density, which according to (18) will therefore be given as a linear function of the cosmic temperature by the order of magnitude estimate

$$\rho \approx \eta^{1/2} T\,\Theta\,, \tag{20}$$

which is interpretable as meaning that the coefficient c in (14) will be given roughly by

$$c \approx \eta^{1/2} T\,. \tag{21}$$

(It is to be remarked that this differs from the corresponding estimate in the analysis of Bucher and Spergel (1999; 1999) by the inclusion of the factor $\eta^{1/2}$, which can be expected, as will be shown below, to have a magnitude comparable with 10^{-10}).

5. ESTIMATION OF THE MEMBRANE TENSION

The kind of model envisaged in the foregoing analysis is illustrated by a modified O(N) model of the type considered (albeit in a different astrophysical context involving symmetry breaking at much higher energy scales) by Kubotani and collaborators (Ishahara *et al.*, 1992), of which the simplest relevant case consists of an ordinary O(3) model for a set of real scalar fields Φ_i , $i = 1, 2, 3$, with an explicit symmetry breaking term proportional to a small parameter ε, as given by a potential energy density term given by an expression of the form

$$V \propto \left(\Sigma_i \Phi_i^2 - \eta^2\right)^2 + \varepsilon \Sigma_i \Phi_i^4 \,. \tag{22}$$

If $\varepsilon < 0$ this potential has six minima, corresponding to six distinct degenerate vacuum states given for different choices of index $j = 1, 2, 3$ by $\Phi_j = \pm\eta/u$ and by $\Phi_i = 0\,, i \neq j$, where u is an order of unity factor given by $u^2 = 1 + \varepsilon = 1 - |\varepsilon|$. On the other hand if $\varepsilon > 0$ there will be eight minima given by taking all the fields to have the same amplitude, $|\Phi_i| = \eta/u$ but with different combinations of positive or negative sign, in terms of an order of unity factor that will be given in this case by $u^2 = 3 + \varepsilon$.

Ordinary three-dimensional space, with Cartesian coordinates x^i, will have a natural periodic map, given by $u\Phi_i = \eta \sin\{\pi\, x^i/2\ell\}$, taking it onto a cube in the configuration space with the calibration adjusted so that the cube sides extend just as far as these minima. This gives a field configuration in which the vacua will arrange themselves as a body-centered cubic lattice in the six-fold case, i.e., when $\varepsilon < 0$, and as a face-centered cubic lattice in the eight-fold case, i.e., when $\varepsilon > 0$. For a given mesh scale ℓ, the energy will be minimizable by a continuous adjustment whereby the vacuum region expands to fill nearly all the interior of each cube of the lattice, leaving only a thin boundary layer characterised by a membrane thickness, δ say, given in order of magnitude by

$$\delta \approx |\varepsilon|^{-1/2}\eta^{-1} \,, \tag{23}$$

so that the corresponding surface energy density and membrane tension will be given by

$$T \approx |\varepsilon|^{1/2}\eta^3 \,. \tag{24}$$

In the six-fold case characterised by $\varepsilon < 0$ there will be string-like boundaries where three membranes meet (and monopole-like junctions where eight such strings meet) which means that the system will be classifiable as of "odd" type, and hence that it does not satisfy the conditions needed for the demonstration of stability referred to above (Battye *et al.*, 2005). The doubtful stability of such a system disqualifies it from being a plausible candidate mechanism for solving the dark energy problem under consideration here (though it does not exclude it from relevance in the much higher energy context of the large scale structure formation considered by Kubotani and collaborators (Den *et al.*, 2000)).

For setting up an effectively rigid solid structure, the positive alternative $\varepsilon > 0$ is more promising, since it provides a 2^N-fold system that will always be of evenly intersecting type, not only when N itself is even, but also when N is odd. In particular, for $N = 3$, one gets an eight-fold system of "even" type in which the variously "colored" vacuum states are given by the different choices of sign for the triplet $\{\Phi_1, \Phi_1, \Phi_1\}$, which can be listed in standard (natural) color notation as a set of four opposing (nowhere neighboring) pairs, namely "black" $\{-, -, -\}$, and "'white" $\{+, +, +)\}$, "red" $\{+, -, -\}$ and "cyan" $\{-, +, +\}$, "green" $\{-, +, -\}$ and "violet" $\{+, -, +\}$, "blue" $\{-, -, +\}$ and "yellow" $\{+, +, -\}$. In this case there will be string-like boundaries where four membranes meet (and monopole-like junctions where six such strings meet) so it is clear that the system will be classifiable as of the "even" type to which the simple rigidity analysis referred to above (Battye *et al.*, 2005) is applicable. Taken together such system of membranes, strings, and monopoles will constitute a dynamically well-behaved rigging structure in the sense (Carter, 1995) described in Section 2. The latter (string and monopole) constituents will become less and less important as the system expands, so the rigging structure will end up by being entirely dominated by the domain wall segments, which would be triangular in the six-fold "odd" case, but will be square in the eight-fold "even" case that is relevant.

The lattice structure obtained in this way would be highly isotropic for large values of N. Even for such a low value as $N = 3$, although its mechanical properties would only be approximately but not exactly isotropic at a local level, the rigging system would still provide an effectively isotropic stress tensor. It might also be possible to obtain mechanically isotropic behavior on scales with magnitude ξ small compared with the Hubble radius H^{-1} but large compared with the mesh scale ℓ in a disordered system of the kind whose possibility is suggested by the five-fold example described in the next paragraph. The observational detectability of the very large-scale granulation on the scale of such a correlation length has recently been examined by Friedland (2003), who has thereby derived limits on η that are remarkably consistent with what has been obtained from the line of reasoning described above.

It is to be remarked that although the simple evenly intersecting systems considered above involved an even number of vacuum configurations, it is also possible to have systems of "even" type having an odd number of vacuum configurations. A noteworthy example of an "even" type system having five distinct vacuum states (related by a discrete pentahedral symmetry) is obtainable from an ordinary $U(1) \times U(1)$ model involving four real scalar fields that combine to form a pair of complex fields

$$\Phi_1 + i\Phi_2 = |\Phi| e^{i\phi}, \quad \Psi_1 + i\Psi_2 = |\Psi| e^{i\psi}, \tag{25}$$

by adding a symmetry breaking term with, let us say, a positive coefficient $\varepsilon < 1$, to the usual quartic potential so that it acquires the form

$$V \propto (|\Phi|^2 - \eta^2)^2 + (|\Psi|^2 - \eta^2)^2 + \varepsilon |\Phi|^2 |\Psi|^2 (\cos(\psi + 2\phi) + \cos(2\psi - \phi)). \tag{26}$$

This has minima with $|\Phi|^2 = |\Psi|^2 = \eta^2/(1 - \varepsilon)$ at vacuum configurations for which the values of the angle pair $\{\phi, \psi\}$ can be listed in terms of arbitrarily ascribed "colors" as the obvious combination $\{\pi, \pi\}$, "green" say, together with $\pm\{\pi/5, 3\pi/5\}$, "red" and "blue" say, and finally $\pm\{3\pi/5, -\pi/5\}$, "violet" and "yellow" say. Unlike the eight-fold system discussed above, this five-fold system is not subject to any restrictions on which pairs of "colors" can be neighbors to each other, whether diagonally across a junction or face-to-face directly across a wall. Indeed all conceivable neighboring "color" relationships are exhibited in the pentahedrically symmetric square "tiling" pattern induced on a two-dimensional plane by taking the phases ϕ and ψ as Cartesian coordinates—which suggests that as well as being able to form a periodic crystalline lattice, such a system also offers the possibility of forming a less regular structure.

The question of irregular generalization of this five-fold square tiling example raises the problem of whether, to color a generically disordered two-dimensional tiling geometry, subject to the condition that only quadruple (crossroads type) junctions are allowed, and that two tile domains of the same "color" can never be neighbors, a maximum of five colors will always be sufficient. This 5 color "even" type tiling problem is to be compared with the corresponding "odd" type tiling problem, for which the condition is that only triple junctions are allowed, in which case it is well known, though not so easy (Horgan, 1993) to prove, that four colors will suffice—with the corollary that four colors would also suffice when the junctions are of quadruple type if the rules were relaxed to allow diagonal (though still not face-to-face) contact between tile domains of the same color.

6. CONCLUSIONS

The kind of frozen (locally static) rigging system described in the preceding section will provide roughly what seems to be needed (Conversi *et al.*, 2004) to account for the cosmological appearance of dark energy provided the microscopic field theory parameters η and ε are such that the corresponding value of the coefficient c in (14), which according to (21) and (23) will be given by

$$c \approx |\varepsilon|^{1/2} \eta^{7/2}, \tag{27}$$

is consistent with the order of magnitude suggested by the observational considerations described in Section 3, namely

$$c \approx 10^{-1} e^4 m_{\rm e} \Theta_{\rm c}^2 \approx 10^{-89}\,, \tag{28}$$

in the Planck units that have been used throughout. Due to the high power of its involvement in (27), the required value for the mass scale η is not very sensitive to uncertainties in c or in ε, and works out to be not so very much smaller that the value obtained from the causal limit assumption used originally by Bucher and Spergel (1999; 1999). The mesh-scale ℓ as given by (18) is even less sensitive, coming out to be larger than the black-body wavelength by a factor of the order of 10^{11}—which is comparable with interstellar distances of a few light years at the present epoch—while the mass scale itself will be given by

$$\eta \approx |\varepsilon|^{-1/7} 10^{-25}\,. \tag{29}$$

It can be seen that this will be significantly, but not enormously smaller than the electron mass, $m_{\rm e} \approx 10^{-22}$, assuming that ε has a value that is much—but not too much—smaller than unity.

The meaning of this is that the relevant membrane forming phase transition would have to have occurred somewhere round about the Kev range, prior to the recombination transition whereby the universe became optically transparent but subsequently to the cosmological epoch of primeval element formation, which is the earliest stage for which it can be claimed that we have detailed observational information. (Its occurrence in such a relatively accessible energy regime encourages me to conjecture that the phase transformation in question may have something to do with the breaking of the approximate isospin symmetry between up and down quarks.)

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